华泰期货
HUATAI FUTURES

Empirical Research on Cross-border Arbitrage of Rubber Futures (Part II)

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Abstract

This paper is the second part of empirical research on cross-border arbitrage of rubber. The first part has already discussed and verified the (1) linear correlation (2) cross correlation (3) excess correlation and its asymmetry between JPX rubber and INE rubber, which provides a solid theoretical support for the feasibility of arbitrage transactions.

This part will further explore the cointegration relationship between the two rubber futures, verify the effectiveness of the long-term equilibrium model, and design three robust statistical arbitrage trading strategies accordingly.

Among them, the Equal Value Allocation strategy provides a calculation method for the ideal position holding ratio. After deducting trading costs, the Sharpe ratio of this strategy reaches 1.35, with an annual return close to 10% and a maximum drawdown of 5.81%, offering the best profitability but with a relatively high difficulty in practical operation.

The Beta Coefficient Allocation strategy compensates for the limitations of the equal value ratio in practical operation, making it more convenient to implement.

The Dynamic Switching Allocation strategy, based on the beta coefficient allocation, is more flexible in capturing changes in the strength of the rubber correlation between JPX and INE caused by shifts in rising or falling market trends, thus optimizing the performance of the strategy.

Catalogue

Charts

Empirical Research

The first part has already verified the correlation between JPX rubber and INE rubber in detail, indicating that the correlation between the two is strong and stable enough, laying a theoretical foundation for digging arbitrage opportunities.

The basic logic of statistical arbitrage is to open position when the price difference is "large enough", and to close position when the subsequent price difference returns to the normal level. And statistical arbitrage strategy can make profits only when the price difference behaves like "Mean-Reversion" at the end. To ensure that, we need to excavate the cointegration relationship between JPX rubber and INE rubber, which means their price difference will keep in long-term equilibrium state.

■ Long-term Equilibrium

In order to accurately find the deviation point of the price difference, we focus on mining the cointegration relationship between the two, from which we can calculate the reasonable price range of the other party through the price of one party, so as to judge whether the current price difference deviates from the equilibrium level.

Ding (2020) believed that cointegration is a stronger relationship definition than correlation or Granger causality, because the structured linear equation generated by cointegration can predict the price of a commodity from the price of other commodities. He also proved the important role of liquidity in price prediction.

Liquidity: It is defined and measured by **effective spread**. This concept was first proposed by Roll (1984), who uses the autocovariance of daily price changes as the estimate of effective spread. And Ding (2020) proposed a revised definition of liquidity based on it:

$$l_t = spread = \begin{cases} 2\sqrt{-Cov(\Delta S_t, \Delta S_{t-1})}, & Cov(\Delta S_t, \Delta S_{t-1}) \leq 0 \\ 0, & Cov(\Delta S_t, \Delta S_{t-1}) > 0 \end{cases}$$

Note: S_t represents the closing price of the trading day t, and $\Delta S_t = S_t - S_{t-1}$

In order to smooth the data, we take the rolling 21 trading days (about one month) as the window to calculate the daily liquidity indicators and normalize the liquidity by subtracting their own mean values and then dividing by their standard deviation, so as to eliminate the dimensional impact of different market sizes. According to Ding (2020), the higher the value, the worse the liquidity of the commodity. The liquidity data for JPX, INE and currency rate ($l_t^{INE}, l_t^{Rate}, l_t^{JPX}$) are calculated and reserved for subsequent analysis.

Cointegration relationship: according to Engle (1987), if the residuals of two non-stationary time series after regression are stable, it means that there is a cointegration relationship between them. Next, we will explore the cointegration relationship between them:

Step 1: Take the logarithm of the closing prices of JPX rubber and INE rubber, and the daily yield can be expressed as the difference between the logarithms of consecutive two trading days.

$$r_t^i = lnP_t^i - lnP_{t-1}^i$$

Step 2: Before the cointegration test, we first conducted the Dick-Fuller Test, and found that the price series were non-stationary. After taking the first order difference, it became stationary. That is to say, the original price series were non-stationary, while the return series were stationary.

Table 1: Dick-Fuller Test | Unit: None

	lnP_t^{INE}	lnP_t^{Rate}	lnP_t^{JPX}
DF	-2.50 (0.11)	-0.60 (0.87)	-1.76 (0.40)

After taking the first order difference

	lnr_t^{INE}	lnr_t^{Rate}	lnr_t^{JPX}
DF	-20.60^{***} (0.00)	-25.65^{***} (0.00)	-28.15^{***} (0.00)

Source: Wind, Huatai Futures Research

Figures in brackets represent P values, and *, * *, * * * represent 10%, 5%, and 1% significance levels respectively.

Step 3: We conduct a series of Granger tests to see whether there is Granger causality among them. The results show that for any pair among INE rubber, Rate and JPX rubber, there is at least one direction of Granger causality for the two series.

Table 2: Granger Test | Unit: None

	INE	$Rate$	JPX
INE	-	3.52^{**} (0.06)	4.94^{***} (0.03)
$Rate$	4.07^{***} (0.04)	-	4.37^{***} (0.04)
JPX	1.53 (0.22)	12.41^{***} (0.00)	-

Source: Wind, Huatai Futures Research

Note: statistics in the table are F-values, the numbers in brackets represent P values, and *, * *, * * * represent 10%, 5%, and 1% significance levels respectively.

Step 4: Establish OLS model to fit cointegration relationship.

$$\ln P_t^{JPX} = \alpha + \beta_1 \ln P_t^{INE} + \beta_2 \ln P_t^{Rate} + \varepsilon_t$$

Add their own liquidity indicators into regression model.

$$\ln P_t^{JPX} = \alpha + \beta_1 \ln P_t^{INE} + \beta_2 \ln P_t^{Rate} + \gamma_1 L_t^{INE} + \gamma_2 L_t^{Rate} + \varepsilon_t$$

Table 3: Regression Result Comparison | Unit: None

	Model without liquidity	**Model with liquidity**
R^2	0.687	0.712
Dick-Fuller Test on Residuals	-2.42 (0.14)	-2.98^{***} (0.04)

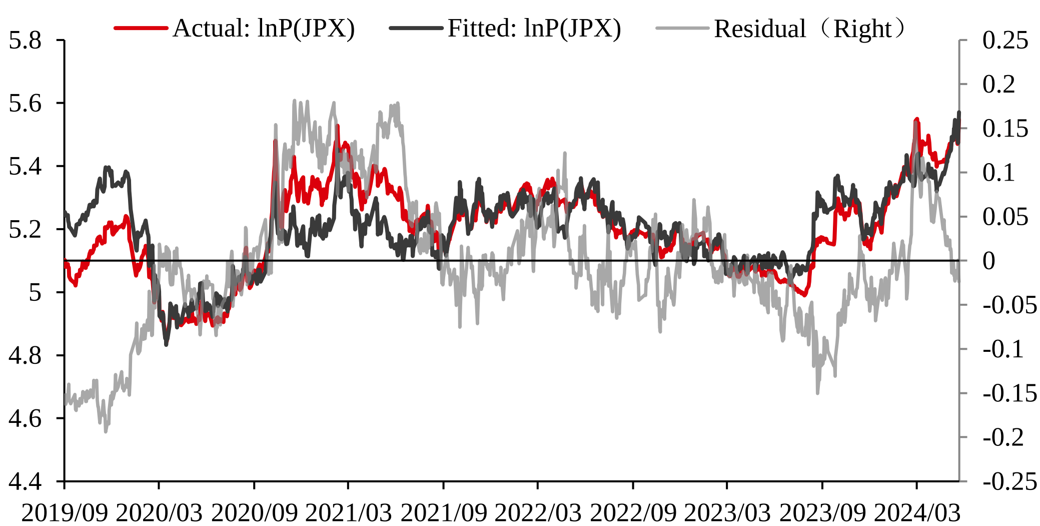
Source: Wind, Huatai Futures Research

Figures in brackets represent P values, and *, * *, * * * represent 10%, 5%, and 1% significance levels respectively.

The residuals after two regressions were tested for stationarity. Table 3 shows that after controlling their liquidity, the residuals of the regression model changed from non-stationarity to stationarity. Therefore, we incorporated the liquidity indicators into the cointegration relationship between JPX rubber and INE rubber to fit their long-term equilibrium model.

$$\ln P_t^{JPX} = -8.892 + 1.273 \ln P_t^{INE} + 0.887 \ln P_t^{Rate} - 0.002 L_t^{INE} + 0.017 L_t^{Rate} + \varepsilon_t$$

Figure 1: Cointegration Relationship Fitted Values | Unit: None



Source: Wind, Huatai Futures Research

Strategy Design

Based on the above cointegration relationship, we can calculate the reasonable range of the price difference between JPX rubber and INE rubber, and timely capture the arbitrage opportunities when the price difference deviates greatly.

■ Basic Logic

When the residual of the regression model deviates from its mean by more than a certain threshold, an arbitrage signal is generated.

When the residual is too high, it means that JPX is overestimated, then we should LONG INE and SHORT JPX.

When the residual is too low, it means that INE is overestimated, then we should LONG JPX and SHORT INE.

■ Parameter Setting

To more precisely define "exceeding a certain threshold," we propose two key parameters to quantify the degree of deviation of the residuals:

(1) Lookback window (X):

number of days included evaluating regression residuals.

(2) Threshold setting (K):

fold of rolling standard deviation.

After determining the combination of parameters, the rules for triggering the opening signal are thereby established:

$\varepsilon_t - rolling\ X\ days\ mean\ > K * rolling\ X\ days\ std$ → LONG INE and SHORT JPX

$\varepsilon_t - rolling\ X\ days\ mean\ < K * rolling\ X\ days\ std$ → LONG JPX and SHORT INE

The easier the opening conditions triggered means that the strategy can capture as many arbitrage opportunities as possible, but at the same time, more transactions will also inevitably bring higher friction cost. On the other hand, the harder the opening conditions triggered, the more opportunities will unfortunately be missed. How to balance between opportunities and costs has become a critical problem.

Later, we will employ grid-search method to find an optimal balance point.

■ Trading Setting

(1) Signal generation time: After the closing of JPX Exchange (14:15 Beijing time), the model residual of the current day is calculated according to the latest closing price. If the residual deviates from a reasonable range, an open signal will be generated.

(2) Trading time: If there is a long/short signal, the position will be opened on the next trading day (T+1) and closed on the day after next trading day (T+2).

(3) Holding period: One day.

(4) Trading price: We use closing price as the trading price and set transaction cost at 0.01% (including slippage and service fee).

(5) Back-test period: November 2019 to June 2024; continuous compounding.

■ Position Allocation

After determining (1) the rules for triggering long and short signals and (2) the trading time for the entry and exit points, we further need to clarify (3) the allocation of position sizes. This will determine how many units of INE rubber contracts should be opened in the opposite direction while opening one unit of JPX rubber contract. The following sections will discuss in detail the application and effectiveness of three different position allocation methods in cross-border arbitrage strategies:

(1) Equal Value Allocation: Based on the principle of equal contract value, the position size is thus inferred.

(2) Beta Coefficient Allocation: The position size is determined based on the beta coefficient of the cointegration relationship.

(3) Dynamic Switching Allocation: The position size is dynamically determined based on the beta coefficient under different states of rising or falling markets.

Equal Value Allocation

Equal Value Allocation is a position size allocation strategy based on the principle of equal contract value. This method collects the latest prices and contract multipliers of JPX and INE rubber contracts, and combines with the exchange rate to derive the equivalent value of the position size ratio. The characteristic of this method is that, regardless of market fluctuations, the daily return of the arbitrage portfolio always reflects the average of the daily returns of JPX and INE rubber contracts.

Taking the closing data on June 7, 2024 as an example, the calculation of position size allocation for the next day's trading is as follows:

Value of 1 lot of JPX contract (in yen)

= Latest closing price of JPX * Contract multiplier

= 357.2 * 5000 = 1,786,000

Value of 1 lot of INE contract (in yen)

= Latest closing price of INE * Contract multiplier * Latest exchange rate

= 13710 * 10 * 21.537 = 2,952,723

Based on the calculation results, the latest position ratio should be adjusted to **JPX: INE = 1: 0.605**.

It is not difficult to see that in real trading, achieving perfect equal-value trading requires stringent conditions, as the contract value will change with the real-time fluctuations of the market and the exchange rates of the two countries, which implies that the strategy exposes to risks not only from the fluctuations in the JPX and INE rubber markets but also from fluctuations in the foreign exchange market.

During the strategy back-testing process, every time after the opening rules were triggered, we calculated the new position size ratios based on the latest prices and exchange rates.

■ Grid Search

After determining all the necessary elements for trading, we use grid-search method to find the optimal parameter combination. We set the lookback window **X to range from 2 to 100 days**, and the threshold value **K to range from 0.1 to 2.0 folds**. For each different combination of parameters, we form a unique strategy and calculate the performance of **Equal Value Allocation** during the same back-test period, then use Sharpe Ratio to select the optimal parameter combination. The results are as follows:

Table 4: Sharpe Ratio of Equal Value Allocation under Different Parameter Combinations | Unit: None

	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2
2	-0.11	-0.14	-0.20	-0.28	-0.36	-0.24	-0.28	-0.21	-0.10	0.10	0.04	-0.02	-0.05	-0.16	-0.08	-0.16	-0.40	-0.50	-0.26	-0.14
3	0.19	0.32	0.22	0.34	0.17	0.08	-0.02	-0.17	-0.23	-0.12	-0.13	-0.03	-0.12	-0.10	0.01	0.16	0.13	0.12	0.19	0.20
5	0.87	0.73	0.55	0.46	0.54	0.54	0.30	0.33	0.22	-0.13	-0.25	-0.43	-0.32	-0.45	-0.34	-0.32	-0.38	-0.40	-0.48	-0.57
10	0.88	0.92	0.81	0.75	0.85	0.86	0.90	0.93	0.88	0.79	0.65	0.53	0.20	0.22	0.28	-0.06	0.11	0.27	0.00	0.06
15	0.70	0.59	0.71	0.67	0.66	0.78	0.79	0.65	0.42	0.60	1.02	0.66	0.45	0.42	0.72	0.89	0.57	0.60	0.21	0.38
20	0.35	0.38	0.57	0.63	0.61	0.83	1.00	0.92	0.84	0.85	0.91	0.82	0.95	0.74	0.66	0.71	0.70	0.82	0.91	0.64
25	0.25	0.36	0.53	0.59	0.60	0.82	0.89	1.09	1.06	1.18	0.98	1.10	0.99	0.71	0.66	0.61	0.65	0.82	0.86	0.66
30	0.21	0.50	0.48	0.45	0.66	0.85	0.71	0.96	1.15	**1.35**	1.17	1.05	0.95	0.86	0.54	0.46	0.70	0.59	0.65	0.59
40	0.13	0.20	0.24	0.32	0.54	0.59	0.64	0.72	0.67	0.97	1.10	1.00	0.84	0.67	0.47	0.54	0.22	0.33	0.57	0.74
50	-0.07	0.02	0.20	0.12	0.14	0.41	0.45	0.51	0.63	0.94	0.77	0.79	0.93	0.95	0.91	0.79	0.39	0.56	0.73	0.59
60	-0.38	-0.19	-0.13	0.14	0.09	0.16	0.15	0.37	0.63	0.64	0.86	0.93	0.62	0.87	1.04	1.00	0.75	0.47	0.61	0.81
70	-0.39	-0.37	-0.27	-0.19	0.12	0.18	0.14	0.11	0.37	0.64	0.68	0.89	1.00	1.06	1.16	1.01	0.80	0.53	0.50	0.33
80	-0.32	-0.34	-0.36	-0.38	-0.25	-0.03	0.28	0.22	0.22	0.67	0.66	0.91	1.15	1.32	1.06	0.95	0.84	0.52	0.33	0.34
90	-0.32	-0.35	-0.38	-0.35	-0.36	-0.11	0.17	0.33	0.34	0.38	0.71	0.97	0.92	1.13	1.29	0.80	0.71	0.90	0.68	0.57
100	-0.34	-0.39	-0.33	-0.14	-0.42	-0.31	0.02	0.34	0.39	0.47	0.62	0.82	0.64	0.83	1.05	0.99	0.64	0.83	0.95	0.74

Source: Wind, Huatai Futures Research

■ **Optimal Strategy**

According to grid search results in Table 4, the optimal parameter combination is 30 days for the lookback window X, and 1-fold for the threshold setting K, with strategy achieving highest Sharpe Ratio.

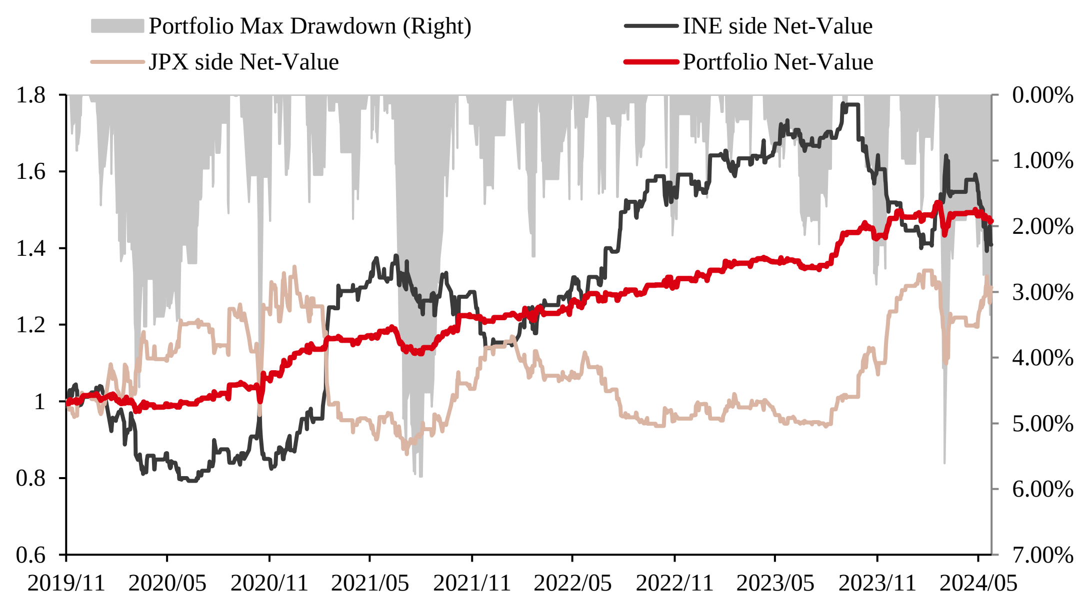
$\varepsilon_t - rolling\ 30\ days\ mean\ > 1 * rolling\ 30\ days\ std$

→ LONG INE and SHORT JPX

$\varepsilon_t - rolling\ 30\ days\ mean\ < 1 * rolling\ 30\ days\ std$

→LONG JPX and SHORT INE

Figure 2: Net-value & Max Drawdown of Equal Value Allocation | Unit: None

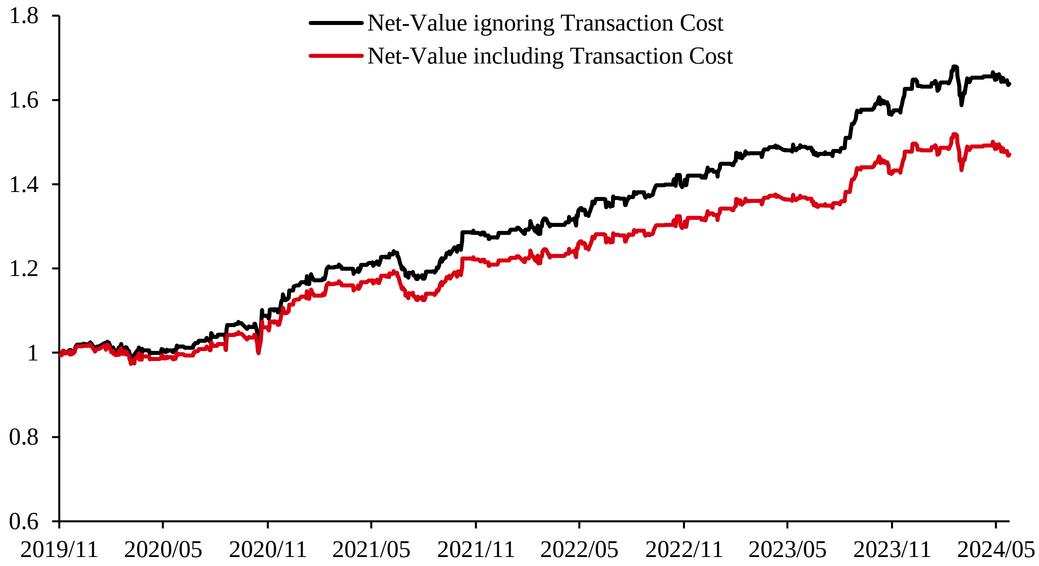


Source: Wind, Huatai Futures Research

In order to have a better understanding of total transaction cost, the black line below restores the potential arbitrage portfolio gains if ignoring transaction costs. By comparing between the black and red line, we can find the total transaction cost is still within an acceptable range.

During the whole back-test period, the opening signal was triggered by 52.1% of days, with the daily profit/loss ratio at 1.36, and the maximum drawdown at about 5%, proving our strategy was kind of robust.

Figure 3: Transaction Cost Impact of Equal Value Allocation | Unit: None



Source: Wind, Huatai Futures Research

Table 5: Equal Value Allocation Arbitrage Strategy Profit and Loss Analysis | Unit: None

	Annual Return	Annual Volatility	Max Drawdown	Max Drawdown Duration Days	Sharpe Ratio	Kalma Ratio
Ignoring Transaction Costs	12.72%	7.31%	5.46%	10	1.74	2.33
Including Transaction Costs	9.83%	7.30%	5.81%	34	1.35	1.69

Source: Wind, Huatai Futures Research

Table 6: Equal Value Allocation Arbitrage Strategy Holding Position Analysis | Unit: None

	Number of Transactions	Ratio of Holding Days	Direction Accuracy	Profit/Loss Ratio
Arbitrage Portfolio	544	52.1%	55.5%	1.36

Source: Wind, Huatai Futures Research

Limitations: Although the equal value allocation method is easy to understand and calculate, it ignores the differences in price sensitivity between contracts in two different markets. Additionally, as the position size ratio calculated based on the equal value principle is often not an integer, which cannot be executed in real trading, the equal-value allocation method, while providing a calculation method for the ideal position ratio, serves only as a theoretical reference for profits and losses. In practical operations, investors need to make appropriate adjustments to the theoretical ratio based on market conditions and the size of their funds.

■ Beta Coefficient Allocation

The beta coefficient allocation method is based on the cointegration relationship between JPX and INE rubber prices, determining the position ratio according to the beta coefficient. This approach considers the price fluctuation differences between futures contracts in different markets and determines relative weights based on their respective volatility sensitivities. While the calculation process is relatively complex and requires high-quality data, with sufficient historical data to support cointegration testing and model fitting, beta coefficient allocation can compensate for the limitations of the equal value allocation method.

Let’s look back at the cointegration relationship formula between JPX rubber and INE rubber proposed in the empirical research section:

$$\ln P_t^{JPX} = -8.892 + \mathbf{1.273} \ln P_t^{INE} + 0.887 \ln P_t^{Rate} - 0.002 L_t^{INE} + 0.017 L_t^{Rate} + \varepsilon_t$$

The meaning of INE's beta coefficient is that, with other variables remaining unchanged, every 1% change in INE rubber price will cause a 1.273% change in JPX rubber price.

If we apply the beta coefficient to the arbitrage strategy, the position allocation ratio should be JPX: INE = 1: 1.273. To address the issue of decimal places, we approximate it as **JPX: INE = 10: 13**, meaning that for every 10-lot position opened in JPX rubber contracts, 13-lot positions are opened in the opposite direction in INE rubber contracts.

■ Grid Search

During the strategy back-testing process, we assume a **total initial capital of 80 million yen**, with **50 million yen in INE and 30 million yen in JPX**, while other settings remain unchanged.

Similarly, using the grid-search method, we set the lookback window **X to range from 2 to 100 days**, and the threshold value **K to range from 0.1 to 2.0 folds**. The Sharpe ratio of the **Beta Coefficient Allocation** arbitrage strategy under each parameter combination is calculated to determine the optimal parameter combination. The results are as follows:

Table 7: Sharpe Ratio of Beta Coefficient Allocation under Different Parameter Combinations | Unit: None

	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2
2	0.01	0.00	-0.02	0.01	-0.01	0.10	0.03	0.08	0.11	0.29	0.31	0.21	0.13	0.10	0.15	0.13	-0.18	-0.20	-0.06	0.07
3	0.47	0.50	0.47	0.41	0.38	0.13	0.10	-0.13	-0.21	-0.19	-0.30	-0.23	-0.13	-0.06	0.04	0.20	0.07	0.09	-0.02	0.14
5	0.93	0.71	0.62	0.63	0.58	0.59	0.43	0.49	0.47	0.11	0.08	-0.21	-0.01	-0.07	-0.02	0.02	-0.06	-0.09	-0.19	-0.22
10	0.53	0.72	0.73	0.68	0.80	0.66	0.65	0.48	0.51	0.54	0.38	0.48	0.34	0.34	0.60	0.16	0.39	0.57	0.13	0.17
15	0.41	0.33	0.43	0.40	0.40	0.68	0.70	0.74	0.47	0.47	0.91	0.72	0.68	0.52	0.77	0.99	0.89	0.65	0.25	0.39
20	0.12	0.12	0.18	0.29	0.21	0.37	0.56	0.55	0.66	0.77	0.97	0.89	0.92	0.85	0.72	0.79	0.67	0.76	1.05	0.83
25	-0.16	-0.04	0.18	0.19	0.29	0.43	0.66	0.77	0.93	1.01	0.87	1.07	1.07	0.76	0.61	0.61	0.49	0.67	0.77	0.56
30	-0.19	0.07	0.11	0.16	0.34	0.59	0.46	0.68	0.91	0.97	0.89	0.87	0.82	0.71	0.43	0.29	0.51	0.35	0.47	0.67
40	-0.14	-0.06	-0.05	-0.08	0.09	0.19	0.18	0.40	0.35	0.61	0.66	0.76	0.59	0.52	0.38	0.61	0.22	0.36	0.62	0.69
50	-0.33	-0.19	-0.02	-0.05	0.02	0.23	0.22	0.26	0.36	0.59	0.52	0.72	0.90	0.85	0.68	0.66	0.42	0.56	0.56	0.52
60	-0.31	-0.22	-0.16	0.11	0.03	0.04	0.08	0.16	0.48	0.37	0.56	0.80	0.58	0.78	0.92	0.89	0.57	0.40	0.58	0.66
70	-0.25	-0.29	-0.30	-0.45	-0.06	0.05	0.13	0.13	0.13	0.41	0.45	0.58	0.86	1.11	**1.28**	1.11	0.66	0.18	0.21	-0.07
80	-0.11	-0.23	-0.25	-0.36	-0.31	-0.10	0.27	0.28	0.34	0.42	0.48	0.80	0.98	1.25	1.08	0.85	0.70	0.11	-0.31	-0.22
90	-0.22	-0.19	-0.17	-0.23	-0.24	-0.09	0.13	0.25	0.38	0.46	0.43	0.74	0.61	0.93	1.16	0.70	0.48	0.72	0.28	-0.06
100	-0.19	-0.17	-0.07	0.03	-0.22	-0.20	-0.08	0.21	0.46	0.50	0.59	0.59	0.50	0.61	1.01	0.91	0.50	0.41	0.61	0.17

Source: Wind, Huatai Futures Research

■ **Optimal Strategy**

Due to the change in allocation rule, the optimal parameter combination calculated by the beta coefficient allocation method differs from the equal value allocation method. As seen from the results in Table 7, when the triggering rule for opening positions is set as X=70, K=1.5, the strategy performs best:

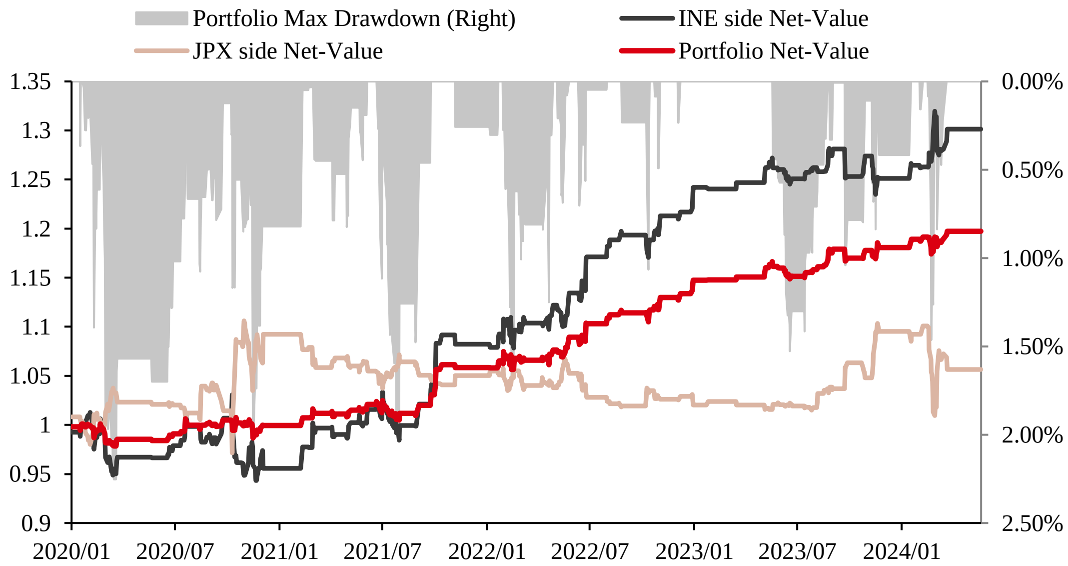
$$\varepsilon_t - rolling\ 70\ days\ mean\ > 1.5 * rolling\ 70\ days\ std$$

→ LONG 13 lots of INE and SHORT 10 lots of JPX

$$\varepsilon_t - rolling\ 70\ days\ mean\ < 1.5 * rolling\ 70\ days\ std$$

→LONG 10 lots of JPX and SHORT 13 lots of INE

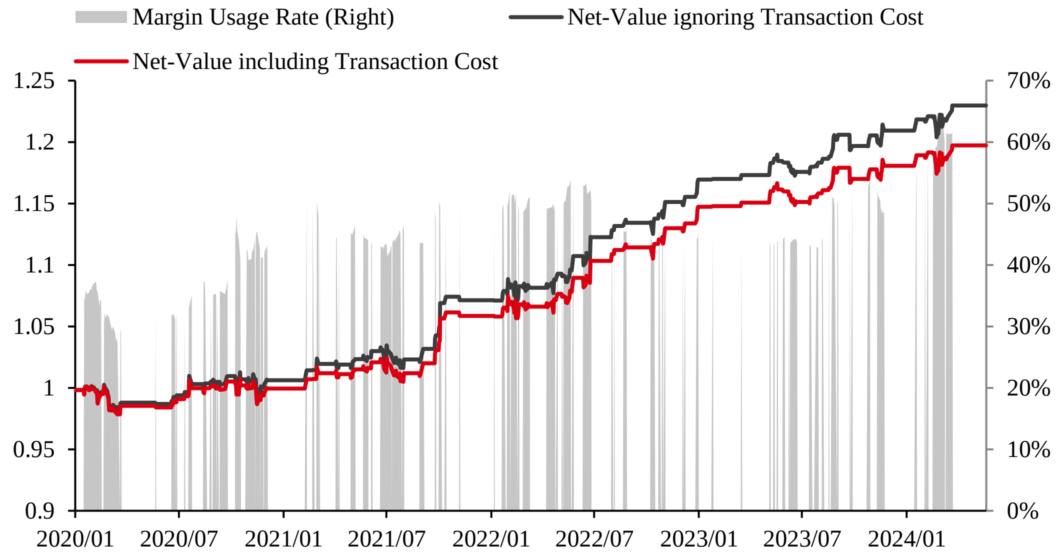
Figure 4: Net-value & Max Drawdown of Beta Coefficient Allocation | Unit: None



Source: Wind, Huatai Futures Research

Due to the stringent conditions for opening positions, the frequency of opening positions is reduced, with only about 30% of trading days triggering the opening signal. The net value curve exhibits phased fluctuations, with the peak margin ratio at approximately 65% and an average of 13.35%. The capital utilization is relatively conservative, and the final Sharpe ratio of the strategy after deducting transaction costs is 1.28.

Figure 5: Transaction Cost & Margin Usage of Beta Coefficient Allocation | Unit: None



Source: Wind, Huatai Futures Research

Table 8: Beta Coefficient Allocation Arbitrage Strategy Profit and Loss Analysis | Unit: None

	Annual Return	Annual Volatility	Max Drawdown	Max Drawdown Duration Days	Sharpe Ratio	Kalma Ratio
Ignoring Transaction Costs	5.33%	3.62%	2.15%	17	1.47	2.49
Including Transaction Costs	4.64%	3.61%	2.25%	37	1.28	2.06

Source: Wind, Huatai Futures Research

Table 9: Beta Coefficient Allocation Arbitrage Strategy Holding Position Analysis | Unit: None

	Number of Transactions	Ratio of Holding Days	Direction Accuracy	Profit/Loss Ratio	Total Cost Ratio	Margin Usage Maximum	Margin Usage Average
Arbitrage Portfolio	301	29.95%	56.19%	1.52	2.68%	13.35%	64.89%

Source: Wind, Huatai Futures Research

Dynamic Switching Allocation

In the previous section on Exceedance Correlation, we have demonstrated that there is no significant difference between the positive and negative exceedance correlation between JPX rubber and INE rubber statistically. However, we also observed that during rolling sample tests, asymmetric price movements occur in certain periods, often with a slightly higher correlation during downward periods compared to upward periods.

Based on this observation, we attempted to introduce dummy variables into the cointegration relationship to distinguish between upward or downward states and analyze whether the degree of influence of INE rubber on JPX rubber differs with state switching.

Original cointegration relationship is:

$$\ln P_t^{JPX} = \alpha + \beta_1 \ln P_t^{INE} + \beta_2 \ln P_t^{Rate} + \gamma_1 L_t^{INE} + \gamma_2 L_t^{Rate} + \varepsilon_t$$

We introduce the dummy variable into the model as both an independent variable and a combined variable:

$$\ln P_t^{JPX} = \alpha + \beta_1 \ln P_t^{INE} + \beta_2 \ln P_t^{Rate} + \gamma_1 L_t^{INE} + \gamma_2 L_t^{Rate} + \boldsymbol{\beta_3 D_t^{INE}} + \boldsymbol{\beta_4 D_t^{INE} * \ln P_t^{INE}} + \varepsilon_t$$

Where D_t^{INE} is the dummy variable, taking a value of 1 when the INE price increases compared to the previous trading day and 0 when it decreases.

By introducing $D_t^{INE} * \ln P_t^{INE}$ as a combined variable into the model, we are allowing the slope of the impact of INE rubber price fluctuations on JPX rubber price fluctuations to vary according to the state of rise or fall.

By introducing D_t^{INE} as an independent variable, we can capture the direct impact of the state switching of INE on JPX.

What needs to be verified is whether the slope and intercept of the model equation change significantly when the dummy variable takes different values:

When the INE price increases, $D_t^{INE}=1$, the slope of $\ln P_t^{INE}$ on $\ln P_t^{JPX}$ is $\boldsymbol{\beta_1}+\boldsymbol{\beta_4}$, and the intercept is $\boldsymbol{\alpha}+\boldsymbol{\beta_3}$;

When the INE price decreases, $D_t^{INE}=0$, the slope of $\ln P_t^{INE}$ on $\ln P_t^{JPX}$ is $\boldsymbol{\beta_1}$,, and the intercept is $\boldsymbol{\alpha}$.

Table 10: Regression Result After Introducing Dummy Variables | Unit: None

	C	$\ln P_t^{INE}$	$\ln P_t^{Rate}$	L_t^{INE}	L_t^{Rate}	D_t^{INE}	$D_t^{INE}*\ln P_t^{INE}$	R^2	ε_t DF
Original Cointegration Formula	-8.892^{***}	1.273^{***}	0.887^{***}	-0.002	0.017^{***}			0.712	-2.98^{**}
	(0.00)	(0.00)	(0.00)	(0.46)	(0.00)				(0.04)
Including Dummy Variables	-8.305^{***}	1.208^{***}	0.889^{***}	-0.002	0.017^{***}	-1.202^{***}	0.133^{***}	0.714	-3.17^{**}
	(0.00)	(0.00)	(0.00)	(0.51)	(0.00)	(0.01)	(0.01)		(0.02)

Source: Wind, Huatai Futures Research

Except for the last two columns, figures in table represent fitting coefficient of OLS model, with figures in brackets representing P values, and *, * *, * * * represent 10%, 5%, and 1% significance levels respectively.

From the fitting results, it can be found that both D_t^{INE} introduced as an independent variable and $D_t^{INE}*\ln P_t^{INE}$ introduced as a combined variable are significant at the 1% significance level, indicating that the different states of rising or falling of INE prices do indeed have a structural impact on the degree of its influence on JPX rubber.

New cointegration relationship formula is:

$$\ln P_t^{JPX}=-8.305+1.208\ln P_t^{INE}+0.889\ln P_t^{Rate}-0.002L_t^{INE}+0.017L_t^{Rate}\\-1.202D_t^{INE}+0.133D_t^{INE}*\ln P_t^{INE}+\varepsilon_t$$

Table 11: Regression Result Comparison | Unit: None

	State	Slope	Intercept
Original Formula	All	**1.273**	-8.892
New Formula	Rising	**1.341**	-9.507
	Falling	**1.208**	-8.305

Source: Wind, Huatai Futures Research

The interpretation of the beta coefficient of INE at this point is: while keeping other variables constant, if the INE rubber price increases by 1% compared to the previous day, it will cause a 1.341% change in the JPX rubber price; if the INE rubber price decreases by 1% compared to the previous day, it will cause a 1.208% change in the JPX rubber price. The smaller slope also confirms the conclusion that the correlation between the two is higher when prices fall.

Applied to the arbitrage strategy, we round the position ratio based on the dynamic beta coefficient. When prices rise, the approximate position ratio is **JPX:INE = 11:15**, and when prices fall, the approximate position ratio is **JPX:INE = 10:12**.

■ Grid Search

The initial capital setting remains the same, totaling 80 million yen, with 50 million yen in INE and 30 million yen in JPX.

Using the same grid-search method, the observation window X was set to a range of 2 to 100 days, and the threshold K was set to a range of 0.1 to 2 times. The Sharpe ratio of the **Dynamic Switching Allocation** arbitrage strategy was calculated for each parameter combination to determine the optimal parameter combination, and the results are as follows:

Table 12: Sharpe Ratio of Dynamic Switching Allocation under Different Parameter Combinations | Unit: None

	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2
2	0.03	0.01	0.00	0.03	0.01	0.12	0.05	0.10	0.14	0.35	0.39	0.30	0.21	0.18	0.21	0.19	-0.14	-0.17	-0.03	0.07
3	0.49	0.53	0.52	0.46	0.41	0.18	0.14	-0.09	-0.17	-0.17	-0.29	-0.20	-0.09	0.02	0.12	0.27	0.15	0.15	0.01	0.17
5	1.01	0.80	0.71	0.72	0.65	0.64	0.51	0.55	0.50	0.15	0.11	-0.19	0.04	-0.05	-0.02	0.04	-0.02	-0.05	-0.14	-0.16
10	0.61	0.82	0.82	0.76	0.90	0.77	0.75	0.55	0.56	0.54	0.40	0.52	0.37	0.38	0.61	0.20	0.43	0.60	0.15	0.20
15	0.49	0.42	0.51	0.48	0.48	0.76	0.79	0.81	0.52	0.49	0.93	0.76	0.68	0.53	0.79	0.97	0.90	0.68	0.30	0.42
20	0.19	0.19	0.23	0.36	0.28	0.43	0.63	0.65	0.72	0.80	0.97	0.89	0.93	0.87	0.75	0.81	0.69	0.74	1.02	0.84
25	-0.08	0.03	0.25	0.25	0.35	0.47	0.72	0.81	0.99	1.06	0.88	1.07	1.07	0.78	0.63	0.62	0.49	0.67	0.75	0.55
30	-0.14	0.13	0.16	0.22	0.40	0.62	0.50	0.75	0.98	1.03	0.92	0.91	0.83	0.73	0.47	0.34	0.51	0.36	0.47	0.65
40	-0.10	-0.02	0.00	-0.04	0.12	0.23	0.25	0.47	0.41	0.68	0.72	0.77	0.60	0.52	0.40	0.60	0.24	0.37	0.59	0.66
50	-0.31	-0.16	0.01	-0.02	0.06	0.27	0.24	0.31	0.42	0.67	0.58	0.73	0.91	0.86	0.71	0.71	0.46	0.60	0.60	0.56
60	-0.29	-0.21	-0.15	0.13	0.05	0.07	0.14	0.22	0.53	0.45	0.61	0.84	0.61	0.79	0.99	0.96	0.66	0.49	0.65	0.74
70	-0.24	-0.27	-0.28	-0.43	-0.04	0.09	0.18	0.20	0.21	0.49	0.52	0.63	0.90	1.18	**1.33**	1.17	0.72	0.30	0.31	0.08
80	-0.13	-0.22	-0.26	-0.34	-0.32	-0.09	0.31	0.35	0.40	0.51	0.60	0.87	1.04	1.32	1.14	0.90	0.77	0.19	-0.22	-0.09
90	-0.24	-0.20	-0.18	-0.24	-0.23	-0.07	0.15	0.31	0.46	0.54	0.52	0.79	0.64	0.96	1.18	0.72	0.51	0.78	0.35	0.02
100	-0.21	-0.17	-0.06	0.02	-0.21	-0.17	-0.04	0.24	0.53	0.56	0.66	0.66	0.51	0.61	1.03	0.94	0.53	0.44	0.70	0.25

Source: Wind, Huatai Futures Research

- **Optimal Strategy**

The optimal parameter combination remained unchanged, with the best performance achieved when the triggering rule for opening positions was set to X=70, K=1.5. However, the position ratio allocation would change according to different state:

$\varepsilon_t - rolling\ 70\ days\ mean\ > 1.5 * rolling\ 70\ days\ std$

→ If INE's price rises → LONG 15 lots of INE and SHORT 11 lots of JPX

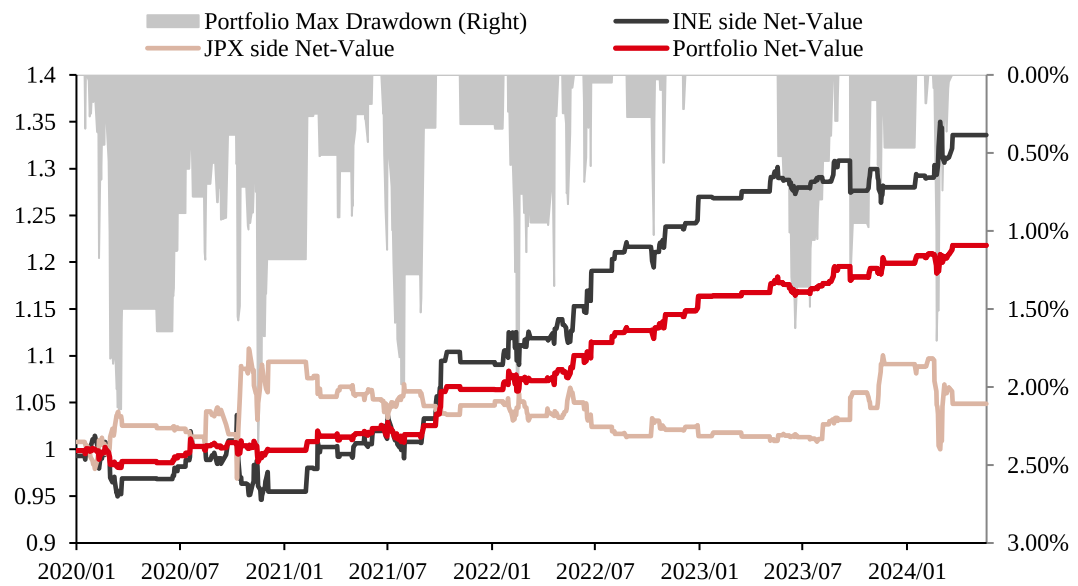
→ If INE's price falls → LONG 12 lots of INE and SHORT 10 lots of JPX

$\varepsilon_t - rolling\ 70\ days\ mean\ < 1.5 * rolling\ 70\ days\ std$

→ If INE's price rises → LONG 11 lots of JPX and SHORT 15 lots of INE

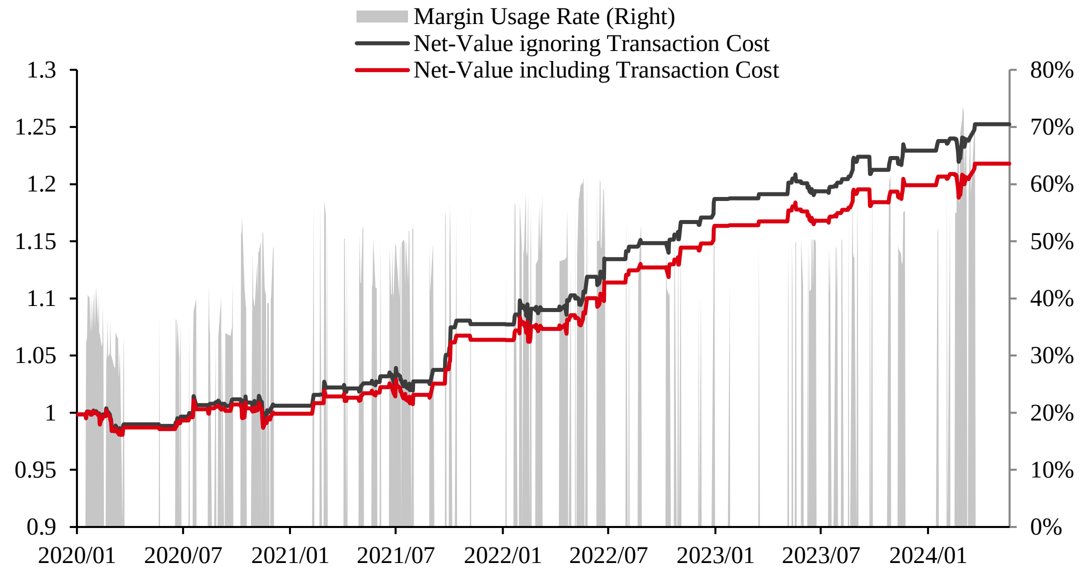
→ If INE's price falls → LONG 10 lots of JPX and SHORT 12 lots of INE

Figure 6: Net-value & Max Drawdown of Dynamic Switching Allocation | Unit: None



Source: Wind, Huatai Futures Research

Figure 7: Transaction Cost & Margin Usage of Dynamic Switching Allocation | Unit: None



Source: Wind, Huatai Futures Research

Table 13: Dynamic Switching Allocation Arbitrage Strategy Profit and Loss Analysis | Unit: None

	Annual Return	Annual Volatility	Max Drawdown	Max Drawdown Duration Days	Sharpe Ratio	Kalma Ratio
Ignoring Transaction Costs	5.81%	3.83%	2.12%	5	1.51	2.74
Including Transaction Costs	5.07%	3.82%	2.38%	74	1.33	2.13

Source: Wind, Huatai Futures Research

Table 14: Dynamic Switching Allocation Arbitrage Strategy Holding Position Analysis | Unit: None

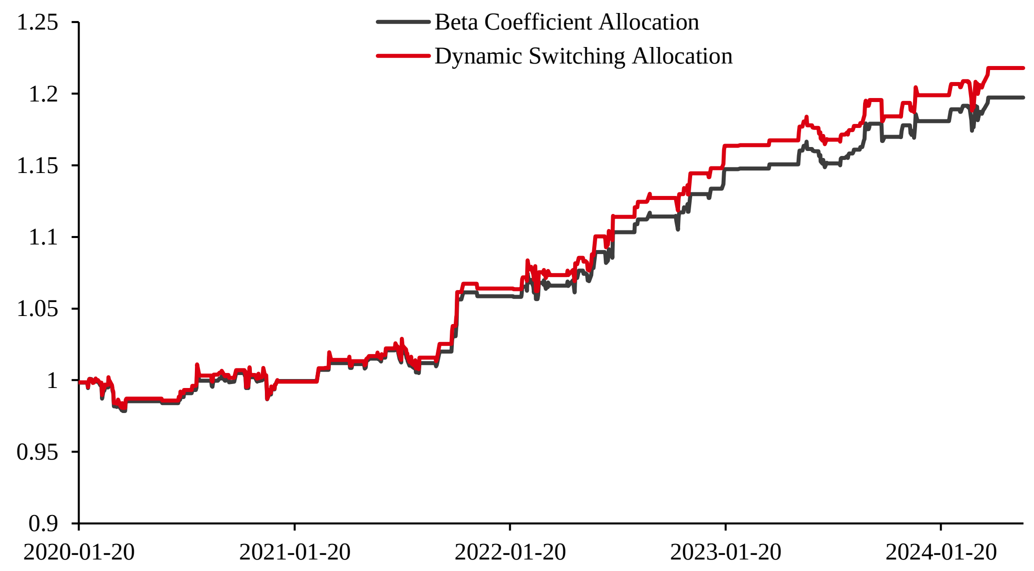
	Number of Transactions	Ratio of Holding Days	Direction Accuracy	Profit/Loss Ratio	Total Cost Ratio	Margin Usage Maximum	Margin Usage Average
Arbitrage Portfolio	301	29.95%	55.18%	1.55	2.80%	13.95%	73.60%

Source: Wind, Huatai Futures Research

Beta Coefficient Allocation VS Dynamic Switching Allocation:

By introducing the dummy variable which distinguishes upward or downward state, we can better capture the real-world correlation between JPX rubber and INE rubber by dynamically switching the position ratio between then, and thus optimize the strategy performance.

Figure 8: Net-value of Beta Coefficient & Dynamic Switching Allocation | Unit: None



Source: Wind, Huatai Futures Research

Table 15: Strategy Performance of Beta Coefficient & Dynamic Switching Allocation | Unit: None

	Annual Return	**Annual Volatility**	**Max Drawdown**	**Max Drawdown Duration Days**	**Sharpe Ratio**	**Kalma Ratio**
Beta Coefficient	4.64%	3.61%	2.25%	37	1.28	2.06
Dynamic Switching	5.07%	3.82%	2.38%	74	1.33	2.13

Source: Wind, Huatai Futures Research

Conclusion

As the second part of the empirical research on cross-border arbitrage of rubber, we verify the effectiveness of the long-term equilibrium model between JPX rubber and INE rubber, and design three robust statistical arbitrage trading strategies accordingly.

Among them, the Equal Value Allocation strategy provides a calculation method for the ideal position holding ratio. After deducting trading costs, the Sharpe ratio of this strategy reaches 1.35, with an annual return close to 10% and a maximum drawdown of 5.81%, offering the best profitability but with a relatively high difficulty in practical operation.

The Beta Coefficient Allocation strategy compensates for the limitations of the equal value ratio in practical operation, making it more convenient to implement.

The Dynamic Switching Allocation strategy, based on the beta coefficient allocation, is more flexible in capturing changes in the strength of the rubber correlation between JPX and INE caused by shifts in rising or falling market trends, thus optimizing the performance of the strategy.

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